Reflection and Transmission of Thermo-Viscoelastic Plane Waves at Liquid-Solid Interface

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***Abstract*- The present paper is aimed at to study the reflection and transmission characteristics of plane waves at liquid-solid interface. The liquid is chosen to be inviscid and the solid half-space is homogeneous isotropic, thermally conducting viscoelastic. Both classical (coupled) and non-classical (generalized) theories of linear thermo-viscoelasticity have been employed to investigate the characteristics of reflected and transmitted waves. Reflection and transmission coefficients are obtained for quasi-longitudinal (qP) wave. The numerical computations of reflection and transmission coefficients are carried out for water-copper structure with the help of Gauss-elimination by using MATLAB software and the results have been presented graphically.**

***Keywords*- Reflection, Transmission, Viscoelastic Solid, Inviscid fluid, Critical angle.**

I. Introduction

The problems of reflection and transmission of waves at an interface between liquid and solid media has many applications in under water acoustics and seismology. Ewing et al. [1], Hunter et al. [2] and Flugge [3] used mathematical models to accommodate the energy dissipation due to viscous effects in vibrating solids. Acharya and Mondal [4] investigated the propagation of Rayleigh surface waves in a Voigt-type [5] viscoelastic solid under the linear theory of non local elasticity. Schoenberg [6], Lockett [7], Cooper and Reiss [8] and Cooper [9] have investigated the problems of reflection and transmission of waves at an interface between viscoelastic isotropic media.

In order to eliminate the paradox of infinite velocity of thermal signals in classical (coupled) thermoelasticity, Lord and Shulman [11] and Green and Lindsay [12] proposed nonclassical (generalized) theories of thermoelasticity which predict a finite speed for heat propagation. Sharma, et al. [13] studied the reflection of piezothermoelastic waves from the charge free and stress free boundary of transversely isotropic half space.

In this paper, we discuss the reflection and transmission of plane waves at the interface between inviscid liquid half-space and thermo-viscoelastic solid half-space. The effects of incident angles and fluid loading on reflection and transmission coefficients are considered. The analytical results so obtained have been verified numerically and are illustrated graphically.

II. Formulation of the Problem

We consider a homogeneous isotropic, thermally conducting, viscoelastic solid in the undeformed state initially at uniform temperature T_0, underlying an inviscid liquid half space. We take the origin of the co-ordinate system (x, y, z) at any point on the plane surface (interface) and the z -axis pointing vertically downward into the solid half space which is thus represented by $z \geq 0$. We choose the x-axis along the direction of wave propagation in such a way that all the particles on the line parallel to the y-axis are equally displaced. Therefore, all the field quantities are independent of y -co-ordinate. Further, the disturbances are assumed to be confined to the neighborhood of the interface $z = 0$ and hence vanish as $z \to \infty$. In the linear theory of homogeneous isotropic, the basic governing field equations of motion and heat conduction for solid and liquid (inviscid) medium, in the absence of heat sources and body forces, are given by

$$\mu^*\nabla^2\vec{u}+(\lambda^*+\mu^*)\nabla\nabla\cdot\vec{u}-\beta^*\nabla(T+t_1\delta_{2k}\dot{T})=\rho\ddot{\vec{u}} \quad (1)$$

$$K\nabla^2 T-\rho C_e(\dot{T}+t_0\ddot{T})=\beta^* T_0\nabla.(\dot{\vec{u}}+t_0\delta_{1k}\ddot{\vec{u}}) \quad (2)$$

$$\lambda_L\nabla\nabla\cdot\vec{u}_L-\beta_L^*\nabla T_L=\rho_L\ddot{\vec{u}}_L \quad (3)$$

$$T_L=-\frac{\beta_L^* T_0^*}{\rho_L C_V^*}\nabla.\vec{u}_L \quad (4)$$

where

$$\lambda^*=\lambda(1+\alpha_0\frac{\partial}{\partial t}), \qquad \mu^*=\mu(1+\alpha_1\frac{\partial}{\partial t}),$$

$$\beta^*=\beta_e(1+\beta_0\frac{\partial}{\partial t}), \qquad \beta_e=(3\lambda+2\mu)\alpha_t,$$

$$\beta_0=(3\lambda\alpha_0+2\mu\alpha_1)\alpha_t/\beta_e, \qquad \beta_L^*=3\lambda_L\alpha^*$$

Here λ, μ are Lame's parameters, α_0 and α_1 are thermo-viscoelastic relaxation times and α_t is the coefficient of linear thermal expansion. ρ is the density of the solid, $T(x,z,t)$ is the temperature change and $\vec{u}(x,z,t)=(u,0,w)$ is the displacement vector; K is the thermal conductivity; C_e is the specific heat at constant

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strain of the solid; t_0 and t_1 are thermal relaxation times ; λ_L is the bulk modulus, ρ_L and α^* are the density and coefficient of volume thermal expansion, $\vec{u}_L$ is the velocity vector and T_L is the temperature deviation in the liquid temperature from ambient temperature T_0^*; δ_{jk} is the Kronecker's delta with $k=1$ for LS theory and $k=2$ for GL theory.

The superposed dot notation is used for time differentiation. To facilitate the solution we define the following dimensionless quantities.

$$x'=\frac{\omega^* x}{c_1}, z'=\frac{\omega^* z}{c_1}, t'=\omega^* t, T'=\frac{T}{T_0}, \sigma'_{ij}=\frac{\sigma_{ij}}{\beta_e T_0}, T'_L=\frac{T_L}{T_0}$$

$$u'=\frac{\rho\omega^* c_1 u}{\beta_e T_0}, w'=\frac{\rho\omega^* c_1 w}{\beta_e T_0}, u'_L=\frac{\rho\omega^* c_1 u_L}{\beta_e T_0}, w'_L=\frac{\rho\omega^* c_1 w_L}{\beta_e T_0}$$

$$\alpha'_1=\omega^*\alpha_1,\ \alpha'_0=\omega^*\alpha_0,\ t'_1=\omega^* t,\ t'_0=\omega^* t_0,\ \bar{\beta}=\frac{\beta_L^*}{\beta_e} \tag{5}$$

where

$$c_3^2=\frac{\mu}{\rho},\ c_1^2=\frac{\lambda+2\mu}{\rho},\ \delta^2=\frac{c_3^2}{c_1^2}, c_L^2=\frac{\lambda_L}{\rho_L}, \delta_L^2=\frac{c_L^2}{c_1^2}$$

$$\omega^*=\frac{C_e(\lambda+2\mu)}{K},\quad \varepsilon=\frac{\beta_e^2 T_0}{\rho C_e(\lambda+\mu)},\quad \varepsilon_L=\frac{\beta_L^{*2} T_0^*}{\rho_L c_v^* \lambda_L}$$

Here ω^* is the characteristic frequency of the solid plate; ε is the thermomechanical coupling constant and c_1, c_2 are respectively, the longitudinal and shear wave velocities in the thermoelastic solid half-space; ε_L is the thermomechanical coupling and c_L is the velocity of sound in the fluid.

Upon using quantities (5) alongwith the relations

$$u=\phi_{,x}+\psi_{,z}\ ,\quad w=\phi_{,z}-\psi_{,x} \tag{6}$$

$$u_L=\phi_{L,x}\ ,\quad w_L=\phi_{L,z} \tag{7}$$

in equations (1)-(4), we get

$$(1+\alpha_1\frac{\partial}{\partial t})\nabla^2\psi-\frac{1}{\delta^2}\ddot{\psi}=0 \tag{8}$$

$$(1+\delta_0\frac{\partial}{\partial t})\nabla^2\phi-\ddot{\phi}=(1+\beta_0\frac{\partial}{\partial t})T \tag{9}$$

$$\nabla^2 T-\dot{T}=\varepsilon(1+\beta_0\frac{\partial}{\partial t})\nabla^2\dot{\phi} \tag{10}$$

$$\nabla^2\phi_L-\frac{\ddot{\phi}_L}{\delta_L^2(1+\varepsilon_L)}=0 \tag{11}$$

$$T_L=-\frac{\varepsilon_L\rho_L}{\bar{\beta}\rho(1+\varepsilon_L)}\ddot{\phi}_L \tag{12}$$

III. Boundary Conditions

The boundary conditions at $z=0$ can be expressed as $\tau_{zz}=-p$, $\tau_{xz}=0$, $w=w_L$, $T_{,z}+H(T-T_L)=0$ (13)
where H is the Biot's heat transfer constant.

IV. Solution of the Problem

We assume wave solutions of the form

$$\{\phi,\psi,T,\phi_L\}=\{A,B,C,D\}\exp\{ik(x\sin\theta-z\cos\theta-ct)\} \tag{14}$$

where $c=\frac{\omega}{k}$ is the non dimensional phase velocity, ω is the frequency and k is the wave number.

Upon using solution (14) in equations (8)-(12), we obtain a system of algebraic equations in unknowns A, B, C and D. The condition for the existence of non-trivial solution of this system of equations upon solving provide us

$$k_j^2=a_j^2\omega^2\quad (j=1,2,3,4) \tag{15}$$

where

$$a_1^2+a_2^2=1-i\omega\delta_0^*\tau_0+i\omega^3\varepsilon\tau'_0\tau_1\beta_0^{*2},$$

$$a_1^2a_2^2=-i\omega\delta_0^*\tau_0,\ a_3^2=\frac{1}{\delta^2}\frac{\delta_0^*}{\alpha_1^*},\ a_4^2=\frac{-i\omega\delta_0^*}{\delta_L^2(1+\varepsilon_L)}$$

$$\tau_0=t_0+i\omega^{-1},\ \tau_1=t_1\delta_{2k}+i\omega^{-1},\ \tau'_0=t_0\delta_{1k}+i\omega^{-1},$$

$$\alpha_0^*=\alpha_0+i\omega^{-1},$$

$$\beta_0^*=\beta_0+i\omega^{-1},\quad \alpha_1^*=\alpha_1+i\omega^{-1}, \delta_0^*=\delta_0+i\omega^{-1}$$

(16)

In the absence of viscous effects $(\alpha_0=0=\alpha_1)$ and thermal field $(T=0=\varepsilon, T_L=0=\varepsilon_L)$, we have

$$a_1^2=1,\ a_2^2=\tau_0,\ a_3^2=\frac{1}{\delta^2},\ a_4^2=\frac{1}{\delta_L^2} \tag{17}$$

A. qP -Wave Incidence Upon a Plane Surface

Let the suffix i and r represent incident and reflected waves, respectively. Omitting the term $\exp(-i\omega t)$, we can write

$$\phi=\phi_t=\sum_{j=1}^{2}A_{rj}\exp\{ik_j(x\sin\theta_j+z\cos\theta_j)\} \tag{18}$$

$$\psi=\psi_t=A_{r_3}\exp\{ik_3(x\sin\theta_3+z\cos\theta_3)\} \tag{19}$$

$$T=T_t=\sum_{j=1}^{2}S_jA_{r_j}\exp\{ik_j(x\sin\theta_j+z\cos\theta_j)\} \tag{20}$$

$$\phi_L=\phi_{Li}+\phi_{Lr}=A_{i_4}\exp\{ik_4(x\sin\theta+z\cos\theta)\}+ A_4\exp\{ik_4(x\sin\theta_4-z\cos\theta_4)\} \tag{21}$$

$$T_L=T_{Li}+T_{Lr}=A_{i_4}\exp\{ik_4(x\sin\theta+z\cos\theta)\}+ S_LA_4\exp\{ik_4(x\sin\theta_4-z\cos\theta_4)\} \tag{22}$$

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where

$$S_j = \beta_0^{*-1}\tau_1^{-1}(a_j^2-1), (j=1,2),\ S_L = \frac{\varepsilon_L \rho_L}{\beta\rho(1+\varepsilon_L)}\omega^2 \tag{23}$$

Upon using equations (18)-(22) in the boundary conditions (14) alongwith the fact that all the waves, incident, reflected and transmitted must be in phase at the interface $z=0$ for all values of x and t, we get

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 = k_4 \sin\theta_4 \tag{24}$$

This with the help of equation (15) implies that

$$a_1 \sin\theta_1 = a_2 \sin\theta_2 = a_3 \sin\theta_3 = a_4 \sin\theta_4 \tag{25}$$

The equation (25) is modified Snell's law in this situation. In the absence of thermal field, viscous effect and liquid, (25) becomes

$$\delta \sin\theta_1 = \sin\theta_3 \Rightarrow \frac{\sin\theta_1}{c_1} = \frac{\sin\theta_3}{c_2} \tag{26}$$

The analytical expression of reflection and transmission coefficients $R_1^{qP} = A_4 / A_{i_1}$ and $T_k^{qP} = A_{r_k} / A_{i_1}\ (k=1,2,3)$ in the presence of thermal field for incident qP wave are obtained as

$$T_1^{qP} = \frac{\Delta_1}{\Delta},\ T_2^{qP} = \frac{\Delta_2}{\Delta},\ T_3^{qP} = \frac{\Delta_3}{\Delta},\ R_1^{qP} = \frac{\Delta_4}{\Delta} \tag{27}$$

where

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ S_1 a_{41} & S_2 a_{42} & 0 & a_{44} \end{vmatrix} \tag{28}$$

and Δ_1, Δ_2, Δ_3, Δ_4 can be obtained from Δ by replacing first, second ,third and fourth column by $\left[-a_{14}\ \ a_{24}\ \ a_{34}\ \ 0\right]'$ respectively.

here $a_{11} = \cos 2\theta_3$, $\quad a_{12} = \cos 2\theta_3$

$a_{13} = -\sin 2\theta_3$, $\quad a_{14} = -\delta^2 \omega_L$

$a_{21} = \alpha_1^* \delta^2 a_1^2 \sin 2\theta_1$, $\quad a_{22} = \alpha_2^* \delta^2 a_2^2 \sin 2\theta_2$

$a_{23} = \delta_0^* \cos 2\theta_3$, $\quad a_{31} = a_1 \cos\theta_1$

$a_{32} = a_2 \cos\theta_2$, $\quad a_{33} = -a_3 \sin\theta_3$,

$a_{34} = a_4 \cos\theta_4$, $\quad a_{41} = i\sqrt{\dfrac{i\omega}{\delta_0^*}}\, a_1 \cos\theta_1 + H$,

$a_{42} = i\sqrt{\dfrac{i\omega}{\delta_0^*}}\, a_2 \cos\theta_2 + H$, $a_{43} = 0$, $a_{44} = -HS_L$,

$$\omega_L = \frac{\rho_L}{\rho\delta^2} \tag{29}$$

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section the reflection and transmission coefficients for qP wave incidence at an interface between thermo-viscoelastic solid and inviscid fluid have been computed numerically. The material chosen for this purpose is Copper, the physical data for which is given by Sharma, et al. [14]

$\varepsilon = 0.00265$, $\quad \lambda = 8.2\times 10^{10}\, Nm^{-2}$,

$\mu = 4.2\times 10^{10}\, Nm^{-2}$, $\quad \rho = 8.950\times 10^3\, kg\, m^{-3}$,

$K = 1.13\times 10^2\, Cal\, m^{-1} s^{-1} K^{-1}$, $\alpha_T = 1.0\times 10^{-8}\, K^{-1}$,

$\alpha_0 = \alpha_1 = 6.8831\times 10^{-13}\, K$, $T_0 = 300K$

The liquid chosen for the purpose of numerical calculations is water, the velocity of sound in which is given by $c_L = 1.5\times 10^3\, m/s$ and density is $\rho_L = 1000\, kg\, m^{-3}$. $T_0^* = 298K$.

Figs. 1 and 2 yields the behaviour of reflection / transmission coefficients for the angles of incidence of longitudinal wave propagates from fluid into solid. It is observed that for longitudinal wave incidence, the reflected longitudinal wave passes through a minimum at critical angles $\theta = 50^0$ for elastic case which is known as Rayleigh-wave angle. At this angle a wave with large surface components is generated. These results parallel those obtained by Mott [10] in the analysis of incidence at a water-stainless steel interface, under the influence of dissipation.

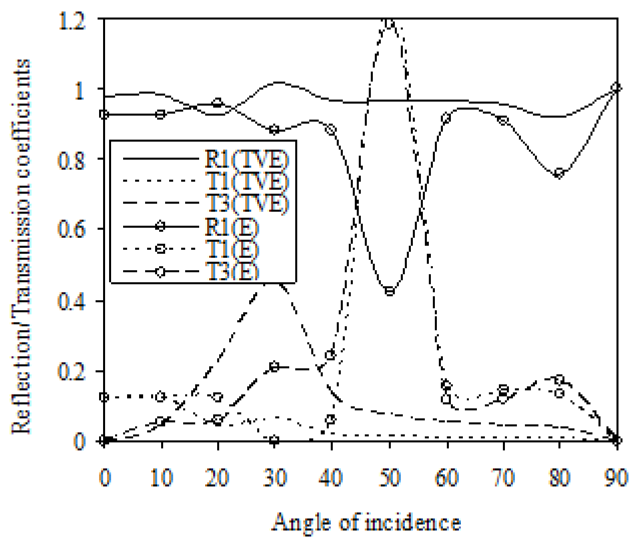

Figure 1. qP -wave incidence at the interface (TVE/E)

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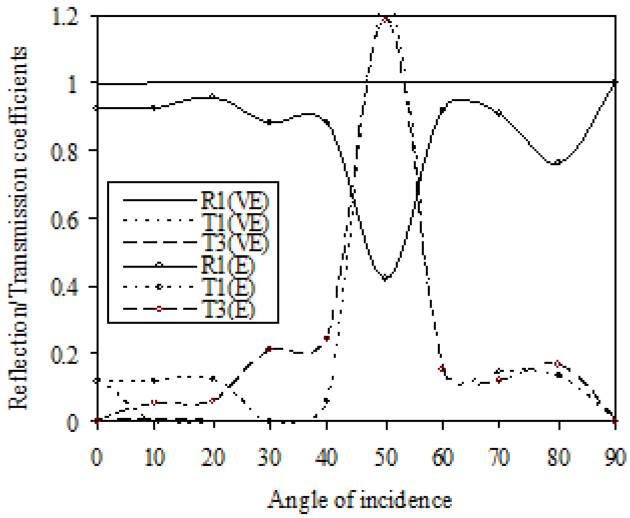

Figure2. qP -wave incidence at the interface (VE/E) in the absence of thermal field

CONCLUSIONS

The reflection and transmission of plane waves at inviscid liquid- thermoviscoelastic solid interface has been analyzed theoretically. The significant effect of incident angle, thermal, viscosity and presence of liquid on the amplitude ratios of reflected and transmitted waves have been observed. Rayleigh angle phenomenon is explained. It is shown that reflected surface wave exist for incidence angles greater than Rayleigh wave angle as explained in [8].

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